

Thinking Sociologically With Mathematics

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A sociological problem that has grabbed my attention is the question of how a sociocultural system's organized activities get simplified enough so that people can learn what they have to do and so that people can reproduce their culture while solving everyday problems creatively. An alternate phrasing of this issue is, How are roles learned and applied in practice?

The standard sociological answer that was offered a third of a century ago was not very satisfying to me as a graduate student. Typically, a role was defined as a set of actions that the role-holder performs in particular circumstances. The set of actions for a role is not written out anywhere, even for the most formal roles, but there was a sense back then that such an inventory could be taken if enough resources were allocated (sort of like a social genome project). I found this approach dissatisfying because it seemed obvious to me that proper role behavior entails performing appropriate actions and also avoiding inappropriate actions, so in essence one would have to learn how each role relates to all possible actions—an impossible learning task. Moreover, the implicit claim was that culture had solved every problem that a role-holder might confront so that a person only had to recall the proper behavior. True, as people solve their immediate problems, their solutions usually are recognizably the same as many others have reached, but sometimes people employ cultural resources and achieve novel solutions.

What I wanted was a way of indexing actions in terms of dimensional positions instead of treating each action as qualitatively distinct. I would assume that individuals, as part of their socialization, learn how their culture distributes actions in the dimensional space. Then the ideal role behavior in a given situation could be defined as a point in the space and the problem of mapping a role to actions is easy: Actions close to the ideal point are role appropriate, and actions far from that point are inappropriate, or deviant. No one needs to learn the actions that are appropriate and inappropriate for a role by rote because an individual can generate as much as needed of the mapping in a particular situation.

I thought that the problem of a role requiring different actions in different circumstances might be handled by shifting the point defining the ideal behavior in some predictable way corresponding to situational variations. For example, the ideal behavior should shift as a role performer interacts with different partners in the role set. The ideal behavior should shift with the status characteristics, personality traits, and moods of the interactants. The ideal behavior should shift when prior acts of deviance need to be repaired.

This is how I understood the problem and the possible solution as a graduate student. I did not have my ideas articulated well, except in a few hand-drawn graphs, and in fact I'm still working on stating the issues and the solutions lucidly. Also, as a graduate student I could not figure out what quantitative dimensions could be used to index actions and provide a basis for identifying ideal points for role behavior. As a matter of fact, as I turned to a dissertation project focusing on affective matters of attitudes and motivations, I began working with the dimensions I would need, but it took some years of post-graduate research before I understood how affect and role definitions fit together. The ultimate result was Affect Control Theory¹.

Modeling Solutions

The three dimensions for indexing actions are: Evaluation, which assesses an action's goodness versus badness; Potency, which assesses an action's significance versus triviality; and Activity, which assesses an action's tumultuousness versus serenity. Studies in more than 30 nations by the psychologist Charles Osgood and his colleagues demonstrated that these three dimensions of affective response are cross-cultural universals. Osgood also demonstrated that judgments with regard to these dimensions can be quantified with bi-polar rating scales. For example, an extremely good action might receive an average rating of +3.0 on a scale pairing good at the positive end of the scale with bad at the negative end.

Raters within a society largely agree in assessing a concept's Evaluation, Potency, and Activity (EPA) on bipolar scales—a fact that now has been demonstrated by research in the U.S.A., Canada, Northern Ireland, Germany, Japan, and China. This indicates that culture determines the EPA connotations of actions, and a particular culture's unique perspective on actions can be examined by plotting actions in the three-dimensional EPA space.

The three-dimensional EPA space allows us to implement the theoretical notion of normative versus deviant actions for a role: actions close to an ideal behavior theoretically are normal actions for a role and actions far from the ideal are deviant. In a minute I will describe how to define a point in the EPA space that corresponds to an ideal role behavior in particular circumstances. Taking such a possibility for granted right now, we can compute Euclidean distances between the ideal point and actions arrayed in the EPA space to assess which actions theoretically are normal and which are deviant.

Here is an example: In traditional American culture the actions of a father interacting with a daughter should be quite good, quite significant, and slightly tranquil (EPA profile of 1.76, 1.64, -0.51). Some actions that are close to this ideal position in the EPA space are thank, explain something to, reassure, instruct, assist, and counsel; such actions presumably are normative actions of a father to daughter in ordinary circumstances. Actions very far from the ideal position should be deviant behavior for a father to a daughter. For instance, the actions nag, beg, ridicule, and heckle are deviant because they are too far from the ideal point in the direction of being bad, petty, and agitated. The actions of rescuing, educating, or curing also may be inappropriate because these actions are too good and significant for the father-daughter relationship ordinarily, though circumstances could arise where such extremely good and potent actions become appropriate. Overall, these predictions from the distance model for normative-deviant action accord with intuitions. Many other examples with similar results have been reported in research publications.

Identifying the position in the EPA space of a role's ideal behavior requires some preliminary understandings. First, from empirical research we know that every role identity has a connotative meaning that can be assessed as an EPA profile. For example, in the U.S.A. a father is quite good, very potent, and a bit reserved (EPA profile of 1.77, 2.14, -0.68); a bully is very bad, somewhat potent, and quite pushy (EPA profile of -2.23, 1.29, 1.86); a daughter is quite good, a bit powerless, and somewhat assertive (EPA profile of 1.73, -0.46, 1.54). Such fundamental sentiments about role identities are provided culturally.

Second, empirical research has yielded equations for predicting the transient impression of a person that results from a particular event, given EPA profiles describing the actor, behavior, and object person before the event occurs. For example, consider the event "The father ridicules the daughter." Prediction equations indicate that the father in this event ends up seeming somewhat bad (-1.29), mainly because his behavior is so bad (EPA profile of -1.96, -0.39, 1.21) and because this bad behavior is directed at a nice, weak person. The daughter ends up losing goodness (going down to an evaluation of 0.56) because she is the object of a nasty act—the standard human response of derogating the victim. In contrast, the prediction equations indicate that "The father assists the daughter" produces a good impression of both the father and the daughter (evaluations of 1.80 and 1.47, respectively). These results were obtained with equations for predicting evaluations; prediction equations also are available to estimate the outcome potency and activity of the actor and the object in events.

Third, a reasonable theoretical supposition is that role enactors behave so as to confirm the meanings of their roles. Especially, people enacting a role try to create impressions of themselves that accord with the role's connotation, thereby maintaining the "expressive order," to use Goffman's term. This means that the actor in a role event should choose an action so that the future goodness, potency, and activity of the interactants as a result of that event will be close to the goodness, potency, and activity of those people's role identities. In other words, an action should be chosen that minimizes the discrepancy between the event-produced transient impressions of the interactants and the fundamental sentiments about the interactants' identities.

Now, as a new event comes into being at a scene, the actor in that event knows the fundamental sentiments about interactants' roles, and knows the impressions of interactants that have formed as a result of past events. The actor can intuit how performing a particular behavior on a particular alter would change current transient impressions to new transient impressions, and through this foresight the actor selects a behavior that will make future transient impressions of self and alter maximally close to the role sentiments for self and alter. This construction process can be modeled as follows. First, we predict the future impressions yielded by the impending event with impression-formation equations. These equations include EPA variables referring to behavior, which are the unknowns in the problem. Second, we define an expression that measures the discrepancy between the predicted future impressions and the fundamental sentiments that are relevant in the event. This expression sums all the squared differences between future EPA impressions and EPA sentiments. Third, we find the behavior that will minimize the discrepancy. That is, we differentiate the expression with respect to unknown behavior variables, set the result equal to zero, and solve for the behavior variables. The result is an EPA profile that defines the position in the EPA space of a role actor's ideal behavior at the given moment.

This approach to defining an ideal behavior shifts the norm as a role performer interacts with different partners in the role set. The different partners have different identities (e.g., the role set of a father includes daughter, son, and mother), these different identities are associated with different EPA sentiments, and those differing sentiments generate different solutions when it comes to matching immediate impressions to fundamental sentiments. A number of research reports have used the modeling solution just presented to demonstrate that ideal behaviors shift in reasonable ways as a role-enactor deals with different members of the role set.

This approach to defining an ideal behavior also shifts the norm as a role performer focuses on repairing prior acts of deviance. Deviant events create peculiar impressions of interactants. Thus after a deviant event a different problem than usual arises as a role-enactor tries to convert current transient impressions into new transient impressions that are maximally close to identity sentiments. For example, a father who suddenly realizes he is ridiculing his daughter might repair this transgression by subsequently soothing, consoling, or cuddling her—behaviors that are much nicer and more serene than usual in order to undo the negative impressions created by ridiculing.

By itself this approach to defining an ideal behavior does *not* shift ideal role behaviors to fit interactants' idiosyncratic status characteristics, personality traits, or moods. For that, we need to expand the model, allowing identities to be modified by personal attributes so that people can take on particularized roles reflecting their individuality as well as their sociocultural positions.

For example, consider the addition of an age-based status characteristic—either old or young—to the identity of father. Calling attention to a father's age makes that particular father deviant to some extent, and the predicted American sentiments about both an old father and a young father are substantially less good and less potent than for an unqualified father (EPA: 1.77, 2.14, -0.68). Of the two age-related qualifications, it is worse to be an old father (EPA: 0.76, 0.50, -2.19) than a young father (EPA: 1.39, 1.01, 1.26). Additionally, an old father is seen as substantially less active than an unqualified father, and a young father is seen as substantially more active.

The overall connotation of an attribute-identity combination is the sentiment that a person has to confirm when the person is maintaining expressive orders associated with both a role and the self. Expected role behavior thereby shifts in the direction of self maintenance.

For example, an old father should act somewhat less good, potent, and lively (EPA: 1.08, 0.71, -1.84) with a daughter than an unqualified father (EPA: 1.76, 1.64, -0.51), which leads to special role actions like cautioning and listening to. The same principle works with personality traits; e.g., an introverted father should act a bit good, a bit reserved, and with little authority (EPA: 1.01, 0.36, -1.20), somewhat like an old father but engaging in low-potency acts. The principle is also the way that moods influence behavior expectations: e.g., an angry father (EPA: -0.37, 1.27, -0.15) with a daughter should act almost affectively neutral (EPA: 0.47, 0.65, -0.09), which promotes actions like scrutinizing the daughter or querying her.

We incorporate attribute-identity combinations into the role-action model by obtaining EPA measures for person modifiers as well as identities. Equations have been derived empirically to predict outcome EPA profiles that result when attributes and identities are combined. Then we use the measured EPA profiles for a

given role and a given attribute to obtain a combination EPA which serves as the fundamental sentiment that has to be confirmed by action. The combination EPA is used in the expression that sums all the squared differences between future EPA impressions and EPA sentiments.

Benefits of Formalization

One benefit of formalizing a theory is that formalization requires a detailed specification that addresses problems that weren't evident prior to formalization. This benefit essentially is an extension of the familiar lesson that one learns about a theory by writing out an exposition of it. However, I'll give an example from the development of Affect Control Theory that illustrates the special benefit of formalization.

Literature on symbolic interactionism guides us toward the theoretical insight that people construct events in order to maintain the meanings of themselves. ACT models this insight by defining an expression that measures the discrepancy between predicted future impressions and the fundamental sentiments that are relevant in an event. The problem arises when we see that the discrepancy expression can expand to include a number of different sentiments. An event creates a new impression of the actor, of the object person, and even of the behavior that the actor directs at the object. Should the discrepancy expression be set up to maintain the meaning of just a focal person in the event, or should it be set up to maintain the meanings of all the event elements? Stated otherwise, do people behave so as to maintain the meanings of themselves, or do they act so as to maintain their understandings generally? I had to take a stance on this issue before I could write out the discrepancy expression that is used in the model. I chose to make humans meaning-maintainers rather than self-maintainers.

Another frequently mentioned benefit of formalization is that one can derive concrete implications to foster empirical studies to evaluate and improve the underlying theory. In particular, a formal model often yields some sort of quantitative result that can be correlated with reality. Here is an example from ACT.

The discrepancy expression applied to a specific event yields a number that quantitatively assesses how much that event deflects meanings away from cultural standards that underlie sentiments. If the number measuring deflection is small, then the event does not distort cultural sentiments, it is the kind of event the actor should want to produce, and the event should be a relatively likely happening. If the number measuring deflection is large, then the event strains cultural sentiments, actors should want to avoid such an event, and the event should be a relatively unlikely happening. When we tested these predictions against people's actual ratings of event likelihoods, we got confirmation of the theory and also some cause to moderate the theory's claims. Events that produced big deflections always were rated as unlikely. Moreover, this was true for events that made interactants seem too nice as well as for events that soiled interactants' identities, supporting the theory of meaning maintenance, and detracting from a theory of self enhancement. On the other hand, the study provided the first systematic evidence that some events that confirm affective meanings nevertheless can be unlikely for cognitive or logical reasons. For instance, "The schoolgirl tickled the gambler" maintains sentiments fairly well, but raters thought the event unlikely, probably because the interactants are not in each other's role sets. Thus affect regulates role behaviors, but cognitive and logical factors also are involved.

For me, the greatest benefit of formalization is the way an explicit model invites tinkering that can lead to expansions in the scope of the theory. Tinkering with the ACT model was of two different kinds: replacing one kind of empirical equation with another; and solving model equations for different unknowns in order to represent diverse social processes. First consider the matter of interchanging empirical equations.

The discrepancy expression that is the core of the ACT model originally measured maintenance of meaning regarding an event's actor, behavior, and object person. However, a setting can be included in the definition of an event, and actions might be chosen to maintain the meaning of the setting along with other event elements. A series of empirical studies parameterized prediction equations that included settings and tested this extension of the theory. Sensible results were obtained, indicating that people can construct events using a simple actor-behavior-object frame, or alternatively people can construct events with the setting being salient and helping to control behavior.

Later we considered a monad as a social unit, allowing that an event might involve just an actor and a self-directed behavior. We empirically parameterized the relevant prediction equations, and substituted these impression-formation equations into the ACT model. This model also produced plausible results. Moreover, application of the ACT model for monads indicated that self-directed acts cannot maintain positive identities effectively, implying that a person with a positive self needs valued social interaction partners as a resource for self confirmation.

Equations for predicting the impressions produced by events might be different when parameterized with data from different cultures. As it turns out, the structure of impression-formation equations is remarkably similar in cross-cultural studies completed so far. However, even variations in the sizes of coefficients seem to make some difference. For example, application of the ACT model with Canadian impression-formation equations gets better results in Canadian situations than using the model with U.S.A. impression-formation equations. This suggests that considerations influencing social interaction are weighted differently across cultures—considerations like retaining current conceptions of actors, or maintaining evaluative consistency between behaviors and interactants' identities. Studies currently are being conducted in Asia to examine further the significance of culture-specific impression formation processes.

Much of the excitement in developing ACT came from solving model equations for different unknowns in order to represent a variety of social processes.

Consider again the discrepancy expression that measures maintenance of meaning regarding an event's actor, behavior, and object person. Initially the actor and object references in this expression were treated as givens at any moment so that the expression could be differentiated with respect to behavior variables, and the result solved for the behavior that would maximally confirm cultural sentiments. However, once the discrepancy expression is written on paper, it is obvious that one can just as well treat the behavior and object references as givens in order to solve for the kind of actor that maximally confirms cultural sentiments. Does it make any sense sociologically to identify the ideal kind of actor who would perform a given action on a given object person? Think of the action as villainous, like robbing someone, and this sounds like the process of labeling deviants. We constructed a model to identify ideal actors, and it did specify appropriate labels for people engaging in deviant actions. For example, the model indicates that a hoodlum, fiend, thug, assailant, outlaw, or crook are among the kinds of actors who would rob a man. Thereby we arrived at the theoretical insight that labeling is based on the same social-psychological processes as role enactment.

Why not treat the actor and behavior as givens and find the ideal kind of object person for the event? It turns out that doing this models a variant of labeling theory in which people get the identities that they seem to deserve considering the predicaments into which they get themselves. For example, applying this model we find that observers may suppose that a youth who gets flunked by a teacher is an ignoramus, a dunce, or a dropout; observers may suppose that someone who gets evicted by a landlord is a sponger, a drunk, a streetwalker, or a homosexual.

Labeling processes make actions understandable by changing the role identities of interactants. Attribution is an alternative kind of reidentification in which person modifiers are used to adjust our perception of a person in order to understand unexpected actions. Basic role identities stay the same, and action is explained by personality or mood instead of a person's social type. Attribution was modeled in ACT by taking the EPA profile for an ideal interactant as an outcome to be produced by combining a modifier with the interactant's original role identity. Then turning to the equations for predicting attribute-identity combinations, we treat the outcome as known, the EPA sentiment for the identity as known, and solve to get the EPA profile of an appropriate attribute. This approach to modeling attribution yielded reasonable results. For example, instead of changing the identity of a youth who gets flunked by a teacher in order to account for the action, the model says we can view the flunkee as a youth who is lazy or apathetic.

Solving attribute-identity equations for a modifier that produces a given outcome provided an ACT model of emotions as well as a model of attribution. In the case of emotions, we use the transient impression of a person in an event as the outcome that has to be reproduced from the person's identity combined with a modifier from the emotion lexicon. This emotion model implies, for example, that a daughter ridiculed by her father will feel flustered, dejected, or embarrassed. Empirical studies show that the ACT model successfully predicts emotions that people report feeling in various situations.

The last benefit of formalization that I want to mention is the opportunity to conduct computer-based analyses, in which large amounts of data are processed with complex equations in order to yield some sort of verifiable result. The verifiable result often is a quantitative indicator like a percentage that can be checked against empirical data. However, ACT simulations result in graphic displays and in verbal outputs that an analyst checks against personal knowledge.

For example, the ACT simulation software draws a picture of each interactant's facial display of emotion during an event. Thereby, analysts can use their sensitivity to faces to determine rapidly whether an interaction is on track or not. The program also produces verbal outputs describing emotions, ideal behaviors, labels, and attributions for interactants. I have presented some of these verbal results in the examples I have been offering, allowing you to use your own knowledge of American culture to assess whether ACT's predictions are plausible or not.

Over the past 25 years, ACT's computer software for running computer-based analyses played a crucial part in weighing theoretic conceptions. Often simulation results were the first evidence that a new theoretical construction explained some aspect of the social world, and the successful simulation then mobilized us to conduct an empirical study to test the new idea. On some other occasions, simulation results indicated that an idea was wrong or excessive.

For example, I tried extending the labeling solution in ACT in order to identify the kind of setting that would be ideal for an event, thinking that this might explain people's movements between scenes. Here is an example result from this model: a prison is the ideal setting for a father disciplining a son. This theoretical solution clearly does not model group processes so I discontinued that line of work. (However, the model might relate to the production of metaphors—e.g., the son exclaims, "This place is like a prison!")

Simulations with the ACT model have revealed that the maintenance of affective meanings motivates proper role behavior. However, simulations also have revealed that actions can be affectively fitting in a relationship and yet not cognitively or logically appropriate. For example, "make love to" can arise as a predicted behavior in simulations of parent-child interactions. "Greets" can be predicted in the middle of an interaction among two friends. Such results force attending to cognitive sociology and narrative theory in order to get a complete picture of how social interactions work.

Conclusions

I can imagine the thrill of commanding new mathematical vehicles that move minds to novel perspectives and carry legions of researchers into unfamiliar territories. However, my mathematical voyage into affect control theory has been made in just a rowboat. The analogy of a rowboat is appropriate in that my use of mathematics required strenuous mental labor from me, and some of the thrills I felt were anxieties about being swamped and in over my head. The analogy also is fitting in that the mathematics that I used—such as Euclidean space or differentiation—were invented long ago, do not challenge contemporary mathematicians, and are widely available to the public, in the form of undergraduate mathematics offerings at every university.

Even mundane mathematical analysis extends reasoning power so that a theoretician can explore ideas in ways that are not possible with words alone. Indeed, the lesson from my experience is that, even if you are not mathematically gifted, you can get yourself a little mathematical craft and punt yourself into unexplored sociology.

1 This paper is for oral presentation, so I do not interrupt its flow with citations and other parenthetical remarks about Affect Control Theory. Relevant references, computer programs, cross-cultural data sets, and mathematical presentations all are available at Affect Control Theory's web site:

<http://www.indiana.edu/~socpsy/ACT/>